Heat transfer enhancement in suspensions of agitated solids

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Theory

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Particle agitation and conductivity

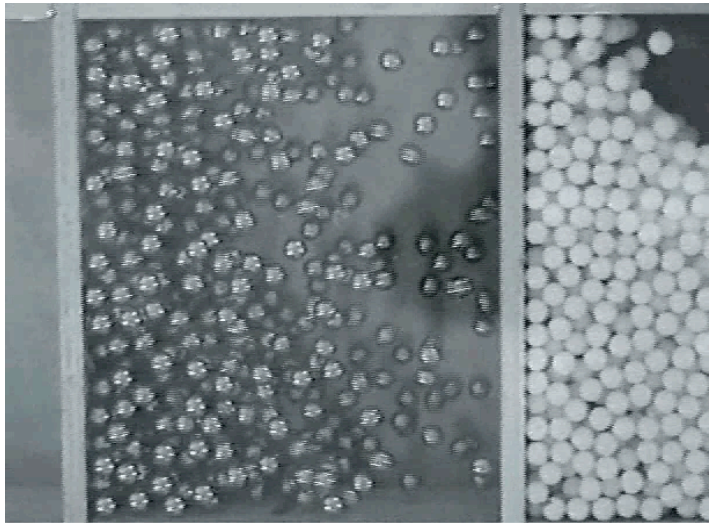
"granular temperature"

$$\Theta = \frac{1}{3} \overline{u'_i \, u'_i}$$

fluctuation velocity u'_i

fluctuation kinetic energy per grain mass

Jenkins, Savage, Lun, others



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Particle agitation and conductivity

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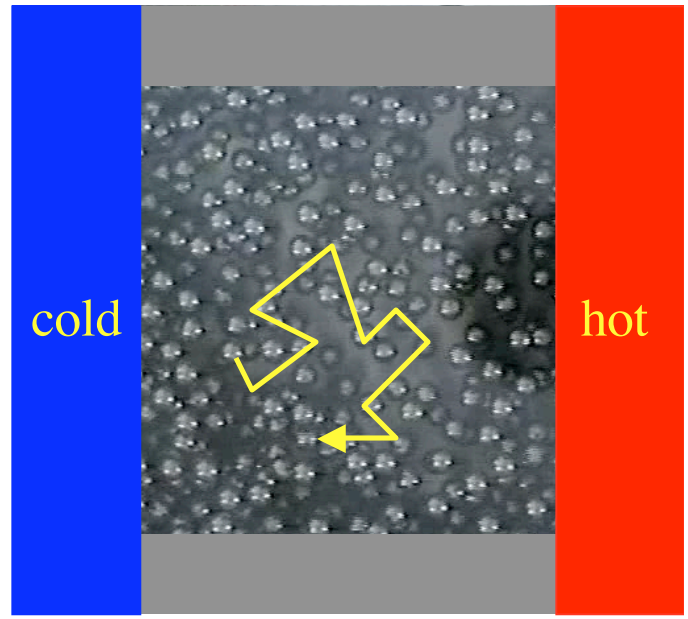
$$\Theta = \frac{1}{3}\overline{u'_i u'_i}$$



Predictions of the kinetic theory

Self-diffusivity $D_s = \dfrac{d\sqrt{\Theta}}{9\sqrt{\pi}\nu g_{12}} \Rightarrow K_s = \rho_s \nu c_s D_s = \dfrac{\rho_s c_s d\sqrt{\Theta}}{9\sqrt{\pi} g_{12}}$

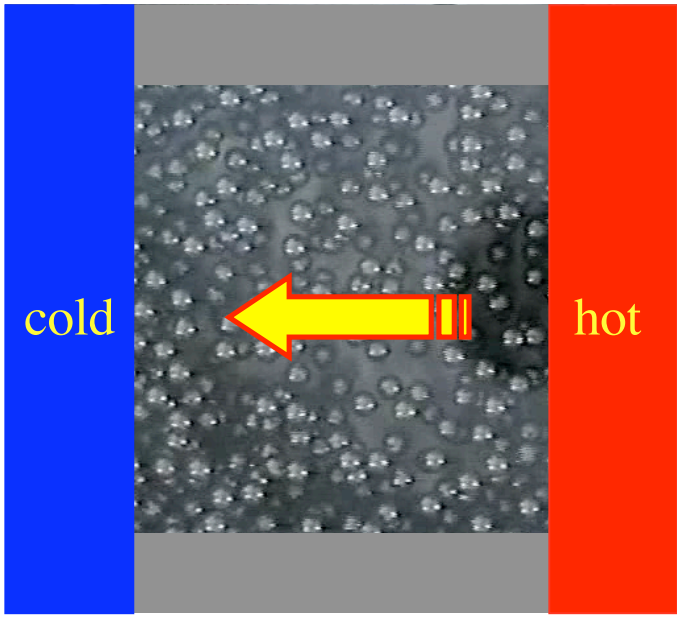
diameter d, solid volume fraction ν, density ρ_s, specific heat c_s , pair correlation g_{12}

Mixture (static) conductivity

Maxwell model (1873)

$$\frac{K_g}{k_g} = \frac{(2+\xi_s) + 2\nu(\xi_s - 1)}{(2+\xi_s) - \nu(\xi_s - 1)}$$

$$\xi_s \equiv \frac{k_s}{k_g}$$



solid volume fraction ν, solid conductivity k_s, gas conductivity k_g

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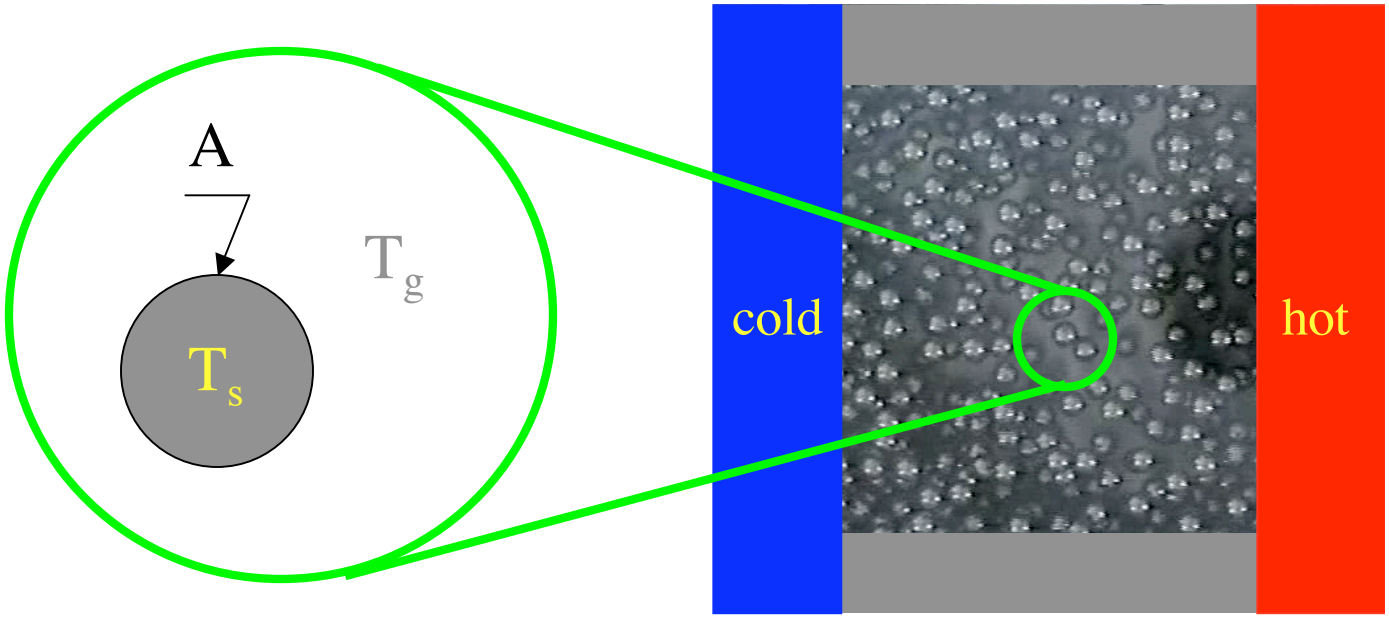
Heat exchange



$$H = nAh(T_s - T_g) = 12\nu \frac{k_g}{d^2}\mathrm{Nu}(T_s - T_g) \qquad \mathrm{Nu} = \frac{h(d/2)}{k_g} \approx 1$$

solid volume fraction ν, number density n, diameter d and surface area A, gas conductivity k_g

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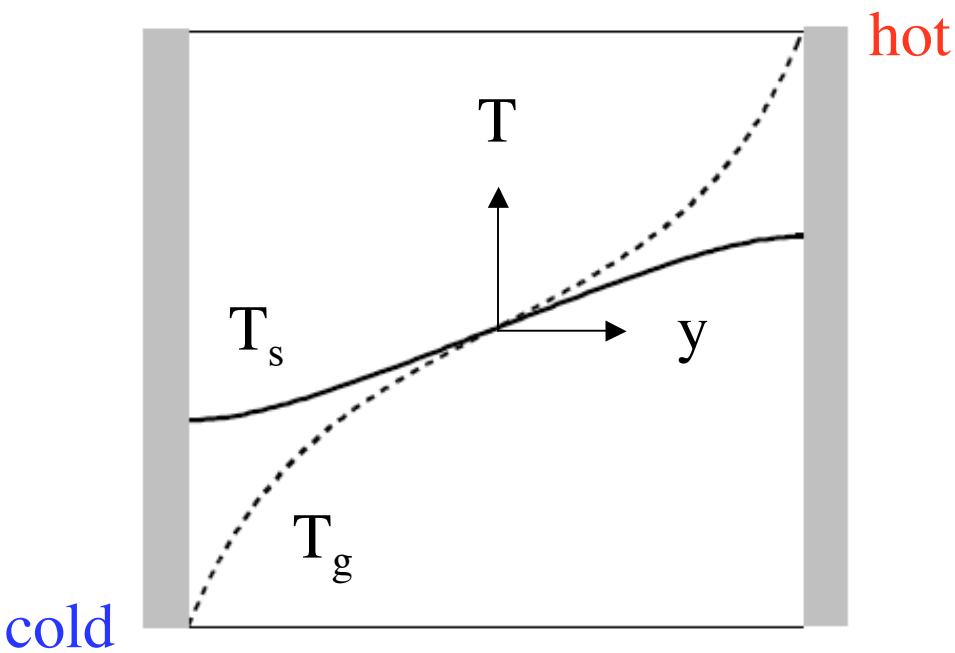
Heat transfer in agitated suspensions

$$\partial/\partial t = 0,\ \partial/\partial x = 0,\ \nabla v = 0,\ \mathrm{v} = 0$$



$$0 = -\frac{d}{dy}\left(-K_g \frac{dT_g}{dy}\right) + H$$

$$0 = -\frac{d}{dy}\left(-K_s \frac{dT_s}{dy}\right) - H$$

phase conductivities K_g and K_s; volumetric exchange rate H

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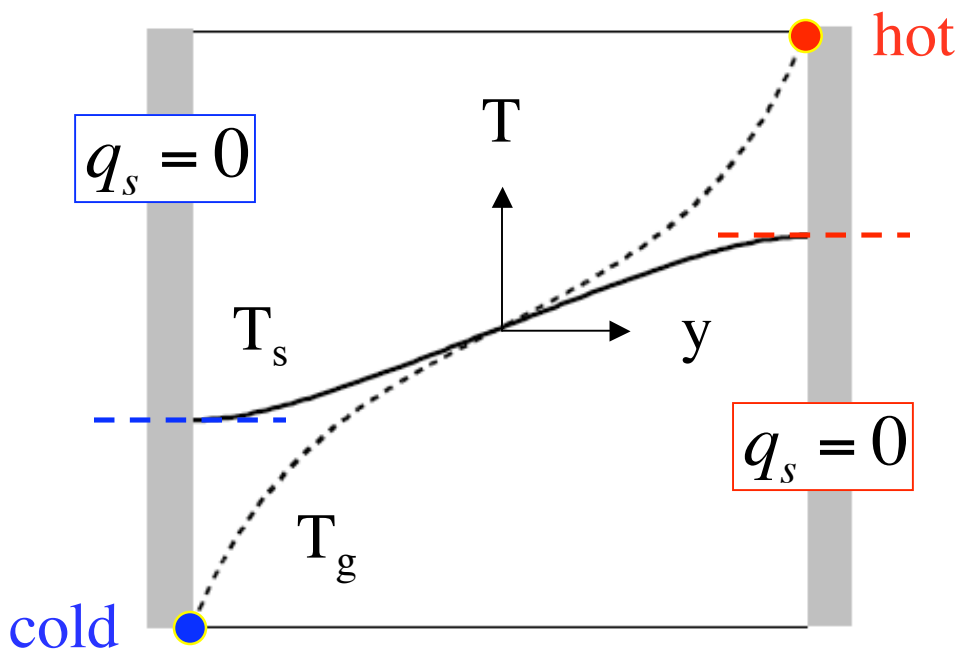
Heat transfer in agitated suspensions

Boundary conditions



Governing equations

$$0 = -\frac{d}{dy}\left(-K_g \frac{dT_g}{dy}\right) + H$$

$$0 = -\frac{d}{dy}\left(-K_s \frac{dT_s}{dy}\right) - H$$

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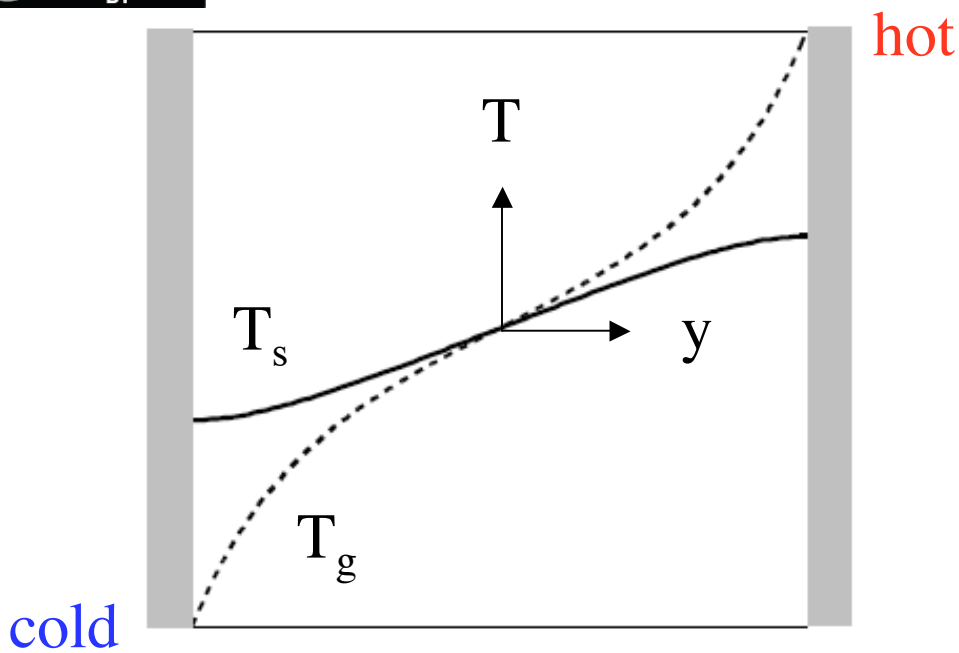
Heat transfer in agitated suspensions

Boundary conditions



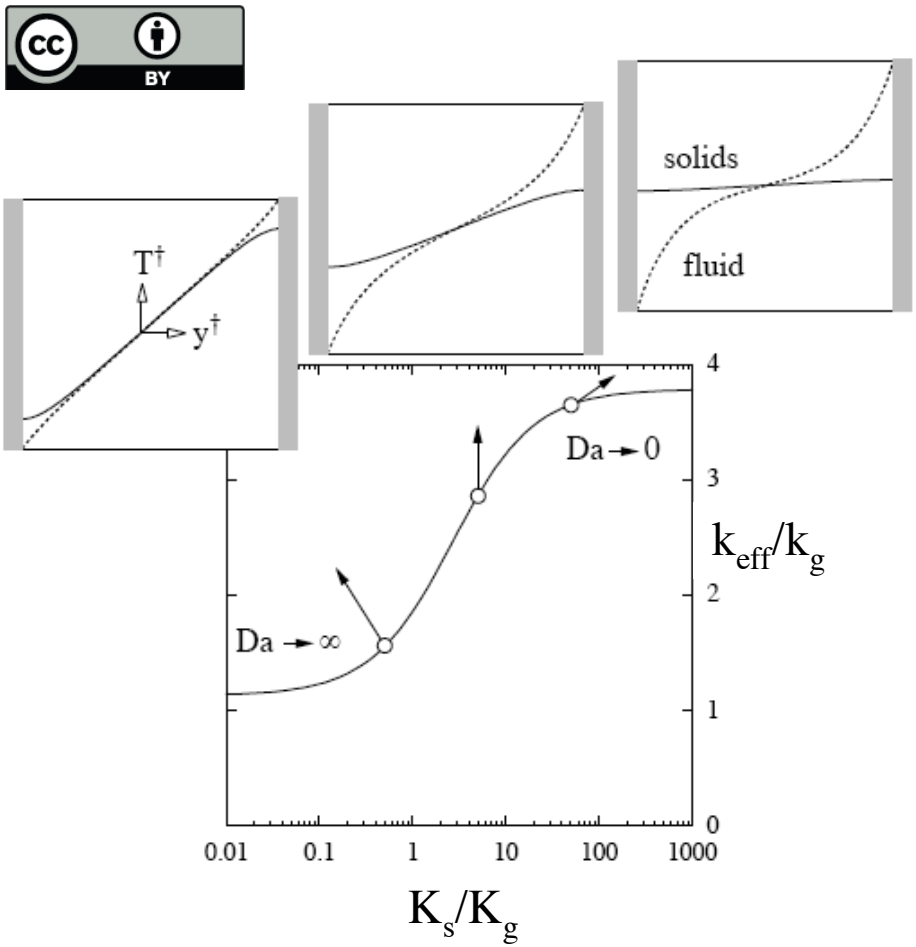
Governing equations

$$0 = -\frac{d}{dy}\left(-K_g \frac{dT_g}{dy}\right) + H$$

$$0 = -\frac{d}{dy}\left(-K_s \frac{dT_s}{dy}\right) - H$$

$$T_g(y) \;\&\; T_s(y) \;\Rightarrow\; q = -K_g \frac{\partial T_g}{\partial y} \;\Rightarrow\; \frac{q(\nu)}{q(\nu = 0)} = \frac{k_{eff}}{k_g}$$

Diffusion and Exchange limits



Conductivity enhancement

$$\frac{k_{\text{eff}}}{k_g} = \left(\frac{K_g}{k_g}\right)\left(\frac{1 + K_s/K_g}{1 + 1/\text{Da}}\right)$$

$$\text{Da} = \frac{(K_g/K_s)(L^\dagger/2)}{\tanh(L^\dagger/2)}$$

$$L^\dagger = \frac{L}{d}\sqrt{12\nu\text{Nu}\left(\frac{k_g}{K_g} + \frac{k_g}{K_s}\right)}$$

phase conductivities K_g and K_s; fluid conductivity k_g; particle diameter d

Complications

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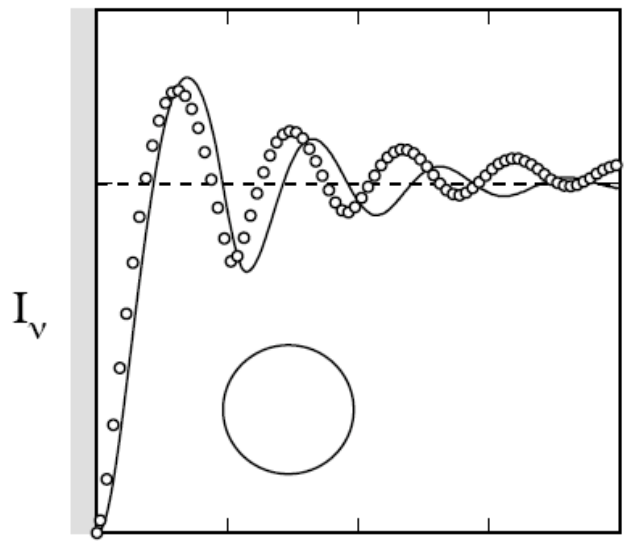
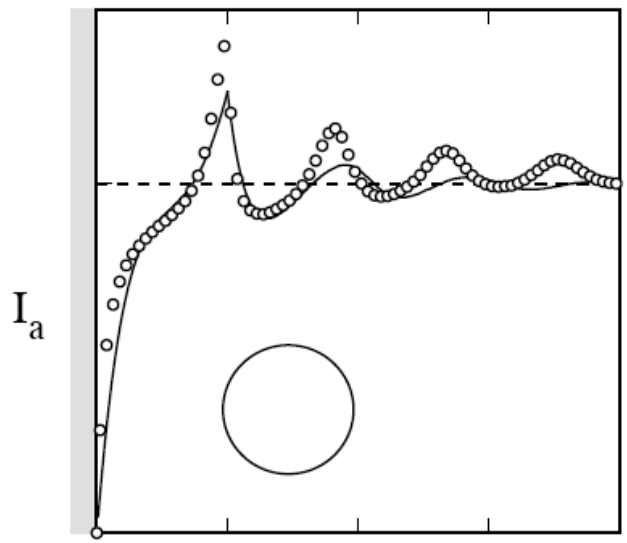
Role of ordering and oscillations

$$\frac{K_g}{k_g} = \frac{(2+\xi_s) + 2\nu(\xi_s - 1)}{(2+\xi_s) - \nu(\xi_s - 1)}$$

$$H = (nA)\, h(T_s - T_g)$$





$$\tilde{\nu} = \nu\, I_\nu$$

$$\widetilde{nA} = \left(\frac{6\nu}{d}\right) I_a$$

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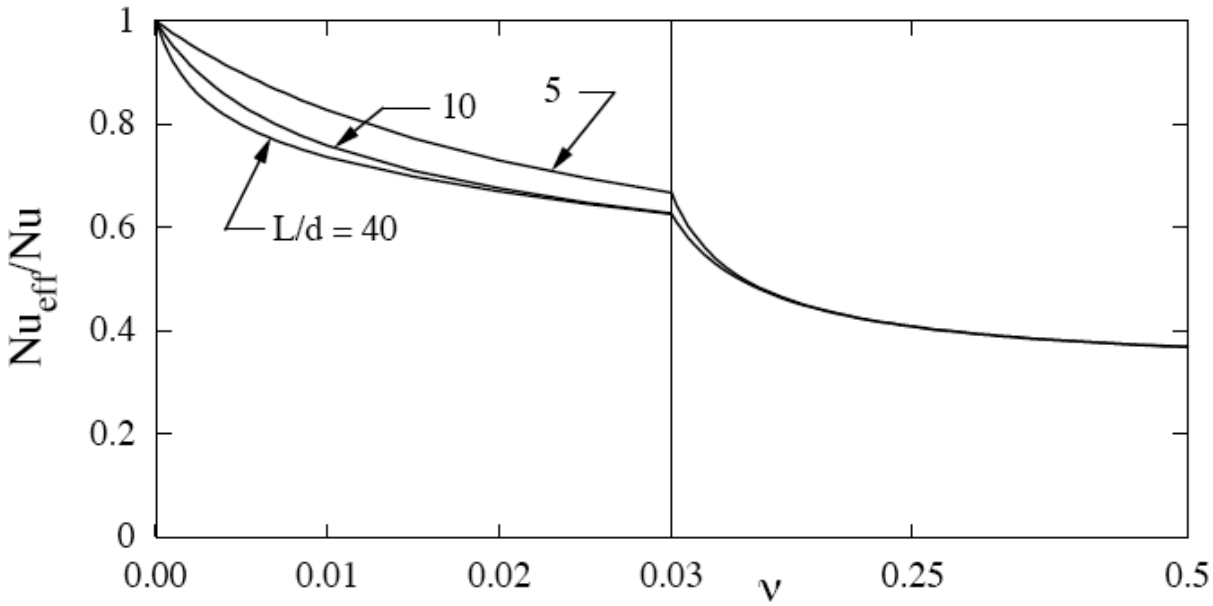
Henderson, Abraham & Barker, Molec. Phys. (1976)

Role of ordering and oscillations

$$\tilde{H} = 12\nu \frac{k_g}{d^2} \mathrm{Nu}_{\mathrm{eff}} (T_s - T_g)$$



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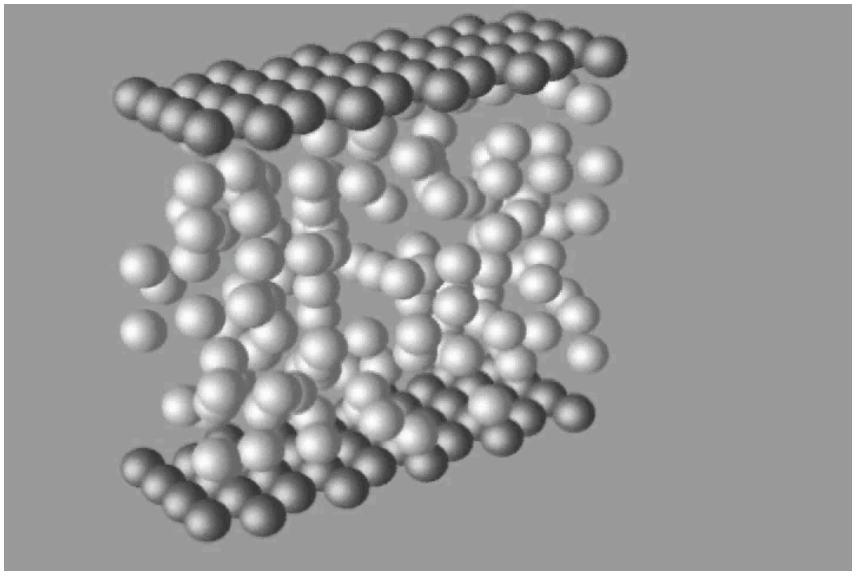
Turbulent enhancements in the exchange limit

Verberg and Koch, Phys. Fluids 2006

$$\overline{u'_i u'_j} = -\omega\, d\sqrt{\Theta}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$



movie: Chris Pelkie

$$\omega \approx 0.04 \text{ at } \nu = 0.3$$

$$k_{gt} = k_g + k_t$$

$$\frac{k_t}{\rho_g c_g} \approx \omega\, d\sqrt{\Theta}$$

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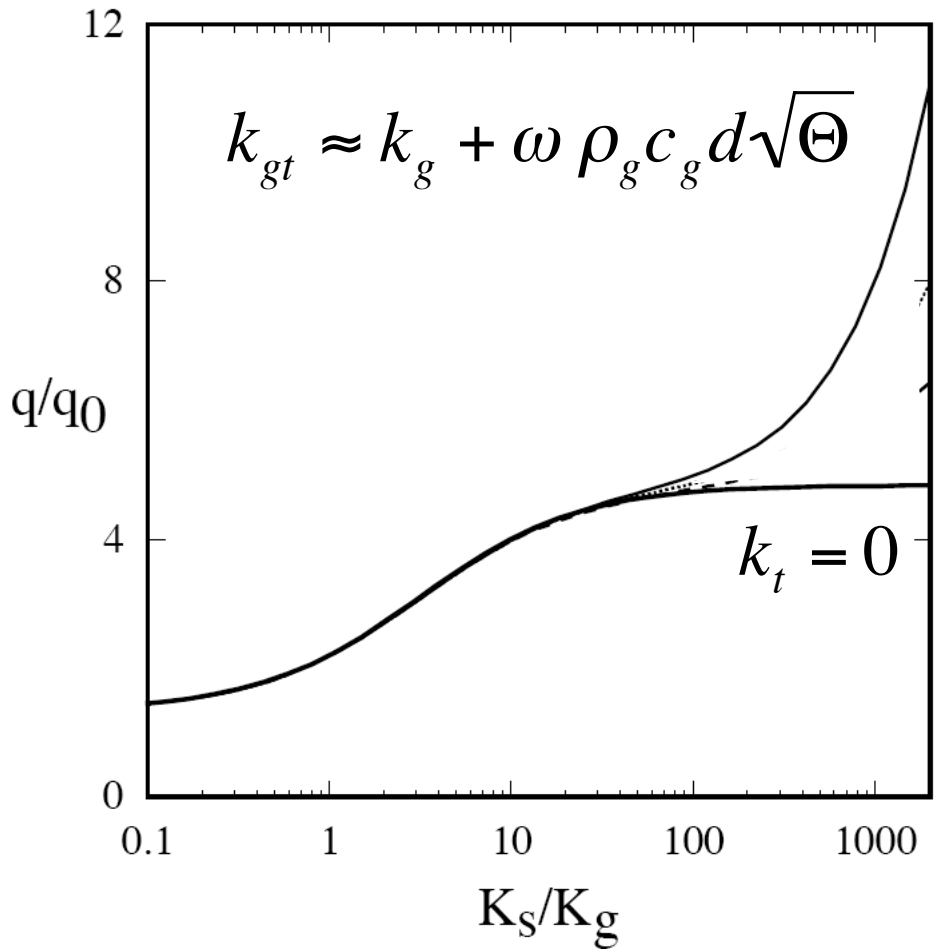
Turbulent enhancements in the exchange limit



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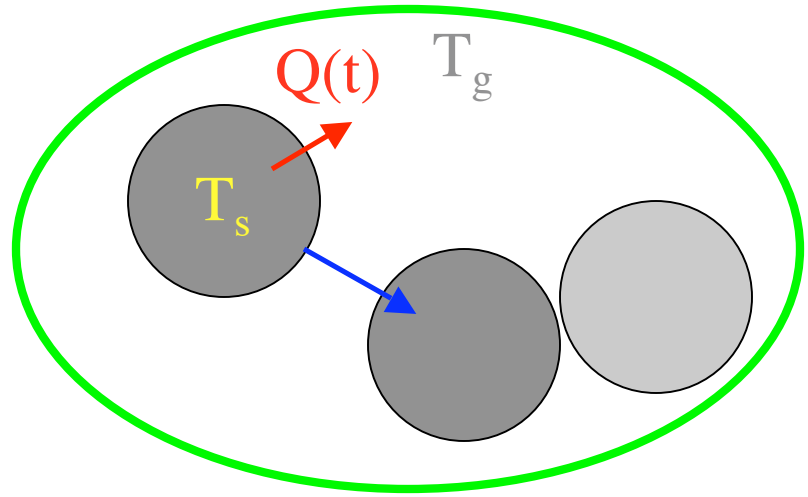
$$\textit{Role of}\ \ \mathrm{Bi} = \frac{h(d/2)}{k_s}$$



Source term $\tilde{H} = nI_a \dfrac{dQ}{dt}$

$$\frac{1}{Q_0}\frac{dQ}{d\mathrm{Fo}} = \sum_{i=1}^{\infty} \frac{6\ \exp(-\xi_i^2 \mathrm{Fo})}{1 - \dfrac{1}{\mathrm{Bi}} + (\dfrac{\xi_i}{\mathrm{Bi}})^2}$$

$$\xi_1^2 = 3\mathrm{Bi}\ [1 - \frac{\mathrm{Bi}}{10} + o(\mathrm{Bi})^2]^2$$

$$\mathrm{Fo} = \frac{4k_s\ \mathrm{t}}{\rho_s c_s d^2}$$

$$t^{-1} \sim f_c = \frac{24}{\sqrt{\pi}} \nu g_{12} \frac{\sqrt{\Theta}}{d}$$ collision frequency

$$\tilde{H} = 12\nu \frac{k_g}{d^2} \mathrm{Nu}(T_g - T_s) I_a \exp(-\xi_1^2 \mathrm{Fo}) \left[1 - \frac{\mathrm{Bi}}{5} + o(\mathrm{Bi}^2)\right]$$

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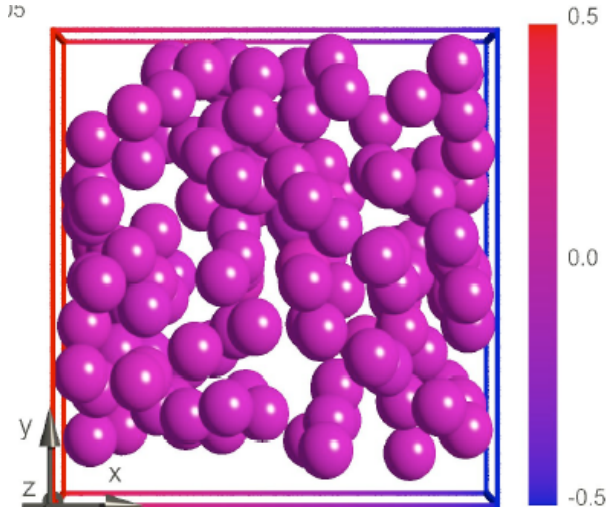
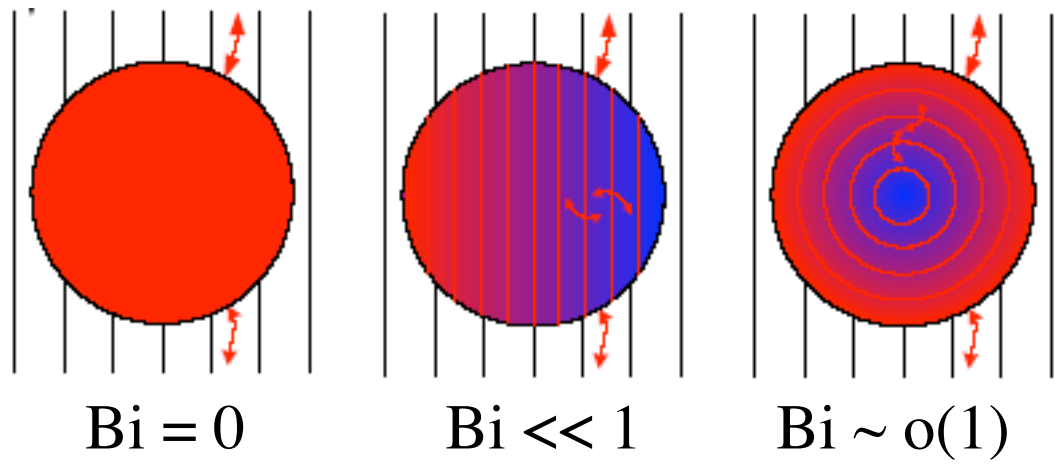
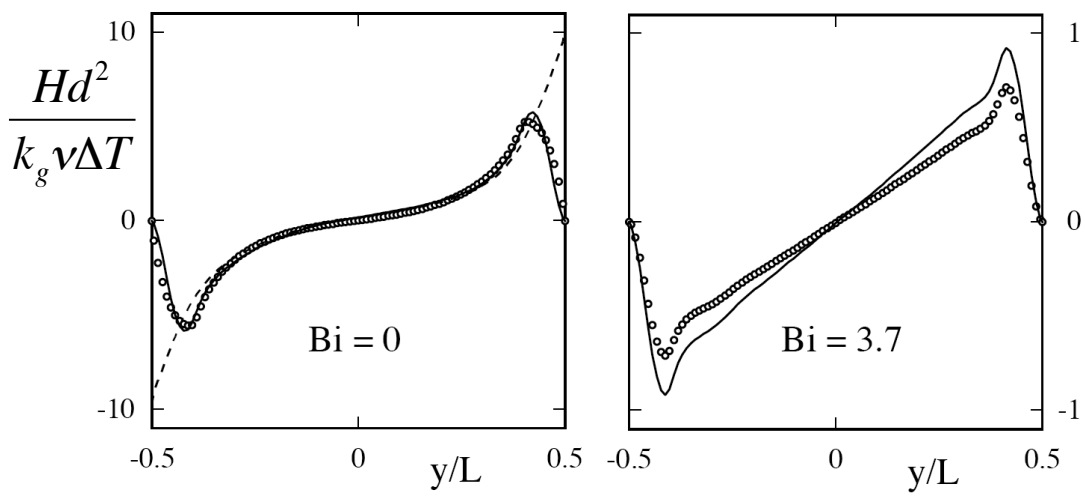
Thermal simulations







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Experiments

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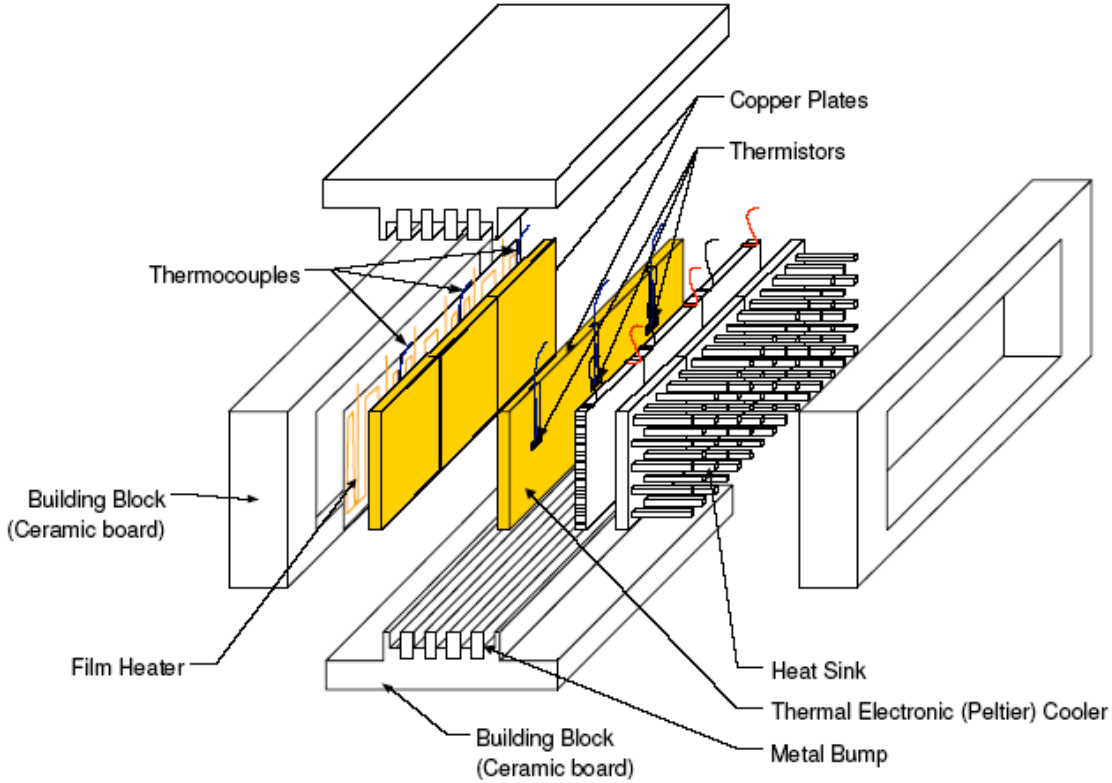
Experiments in the exchange limit



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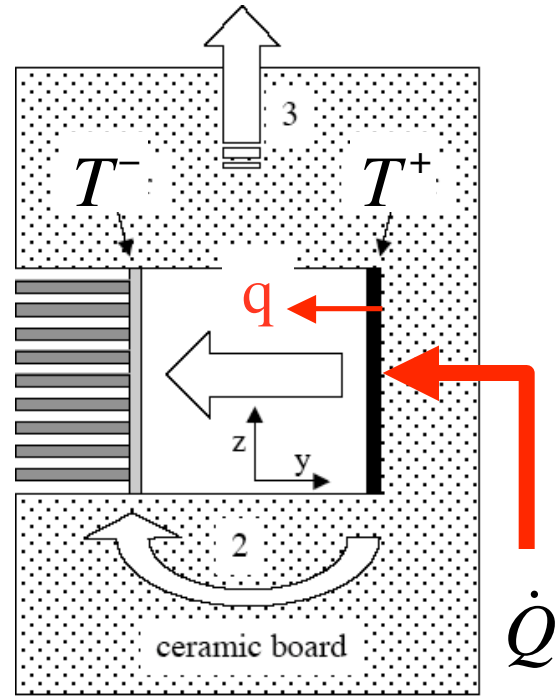
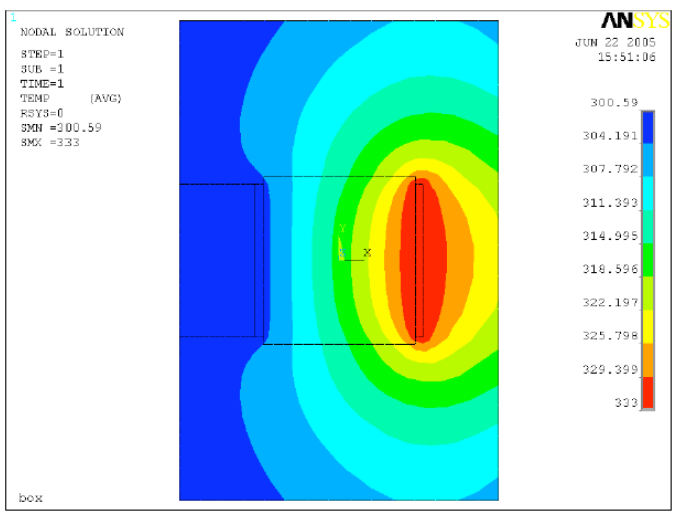
$$\textit{Measure}\ \ k_{\text{eff}} = \frac{qL}{T^{+} - T^{-}}$$

$$\dot{Q} = Aq + \frac{T^{+} - T^{-}}{R_b} + h_b S_b \left(\frac{T^{+} + T^{-}}{2} - T_\infty \right)$$

$$\dot{Q} = f(T^{+}) - \Omega T^{-}$$





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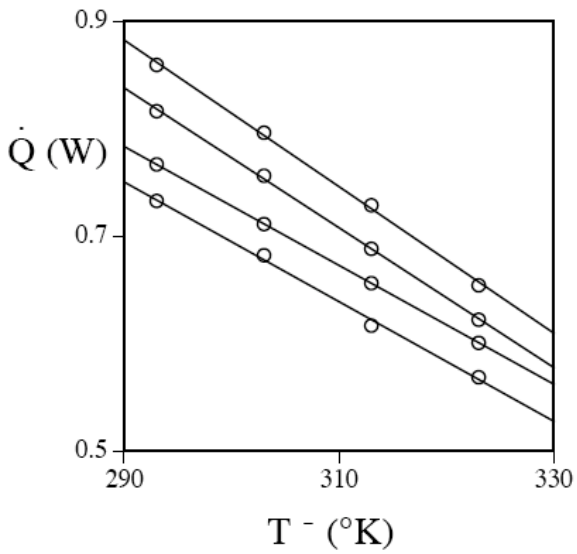
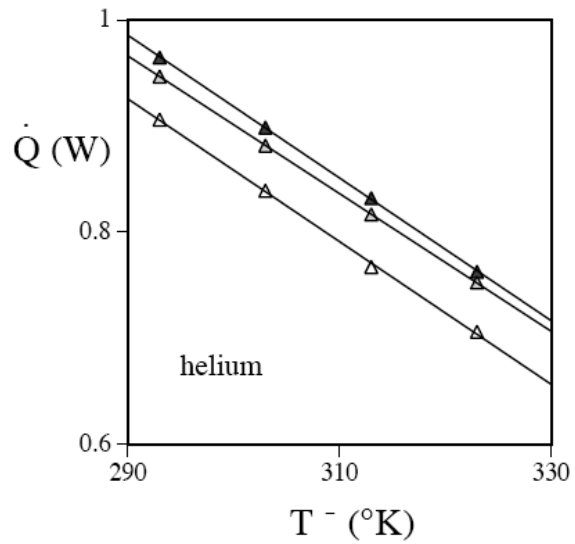
$$\textit{Measure}\quad k_{\text{eff}} = \frac{qL}{T^{+} - T^{-}}$$

$$\Omega = k_{\text{eff}}\frac{A}{L} - \frac{h_b S_b}{2} + \frac{1}{R_b} \qquad \Omega_{\text{He}} = k_{\text{He}}\frac{A}{L} - \frac{h_b S_b}{2} + \frac{1}{R_b}$$





$$\frac{q}{q_0} = \frac{k_{\text{eff}}}{k_g} = \frac{k_{\text{He}}}{k_{\text{air}}} + (\Omega - \Omega_{\text{He}})\frac{L}{k_{\text{air}} A}$$

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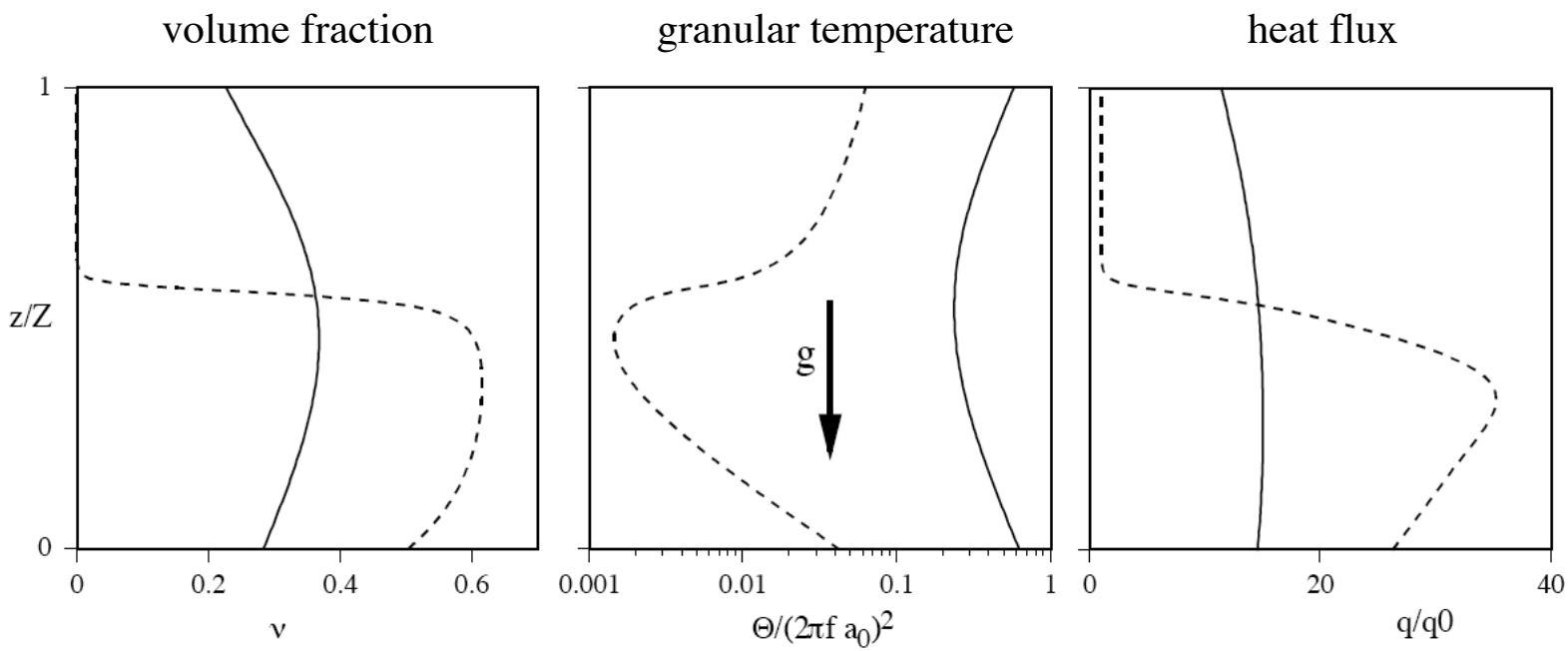
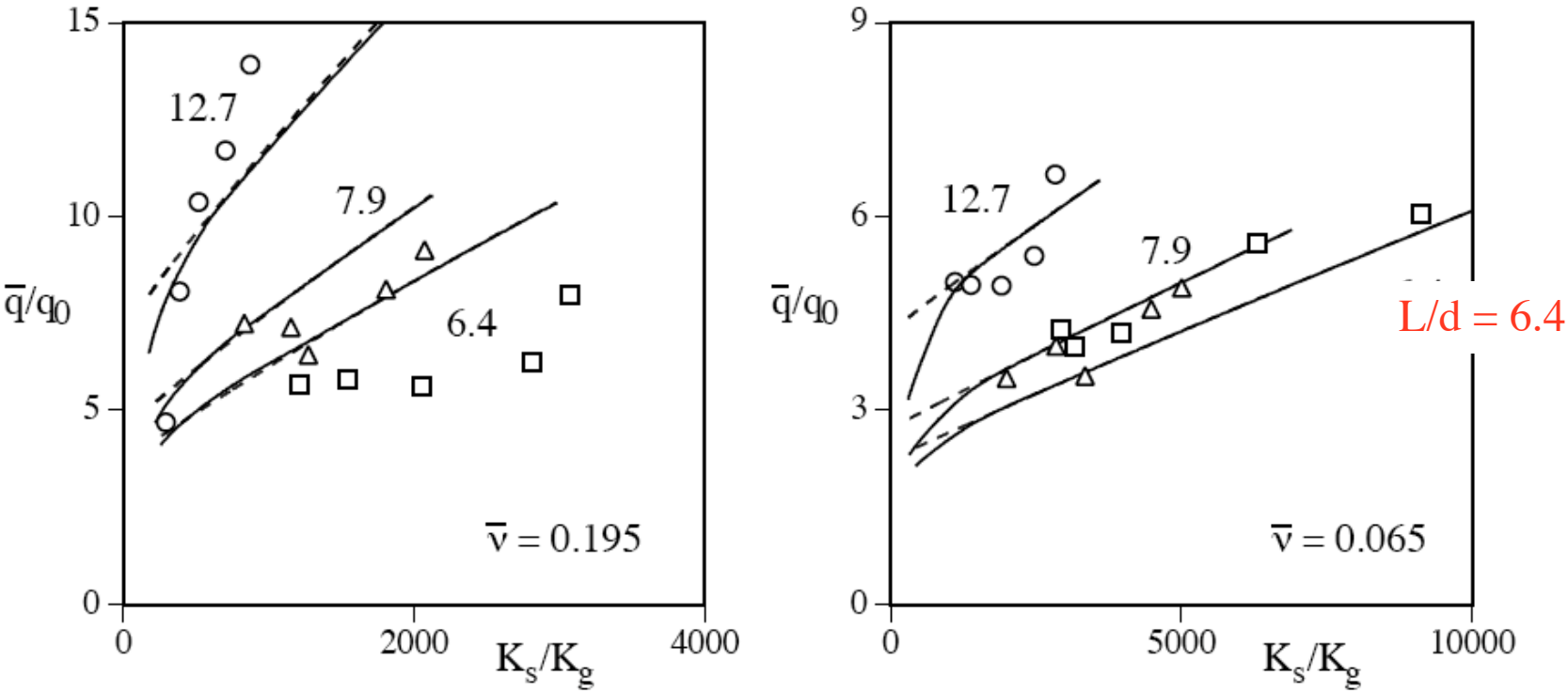
Model the vibrated box



$$\frac{K_s}{K_g} = \left(\frac{\rho_s c_s d \sqrt{gd}}{k_g} \right) \frac{\Theta / \sqrt{gd}}{9\sqrt{\pi}\, g_{12} f_M(\nu;\xi_s)}$$

$$\bar{q} = \int_0^Z q(z)\,dz$$

Role of size in the exchange limit



4mm, 3.2mm, and 2 mm acrylic spheres in air

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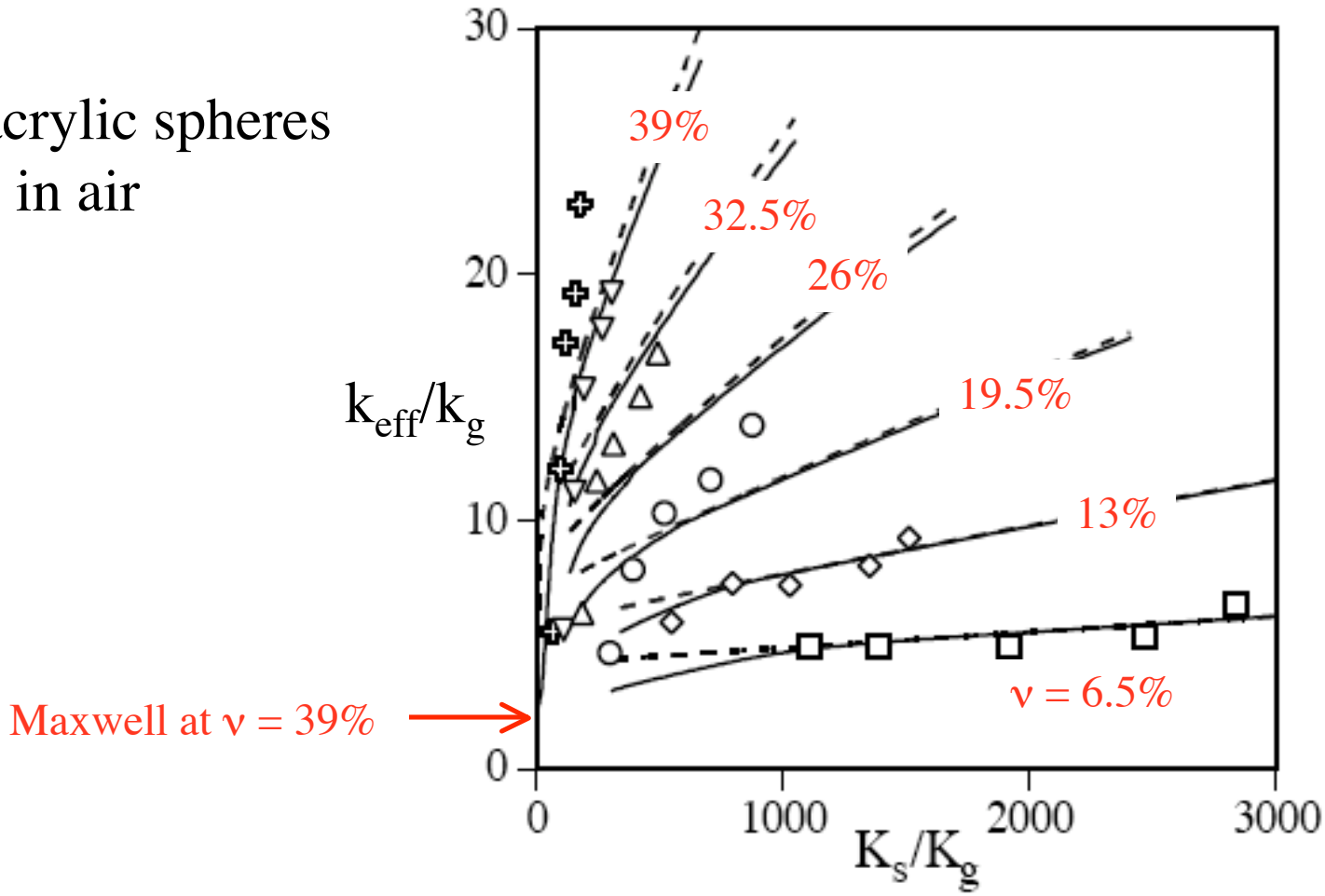
Role of volume fraction in the exchange limit

2 mm acrylic spheres
in air



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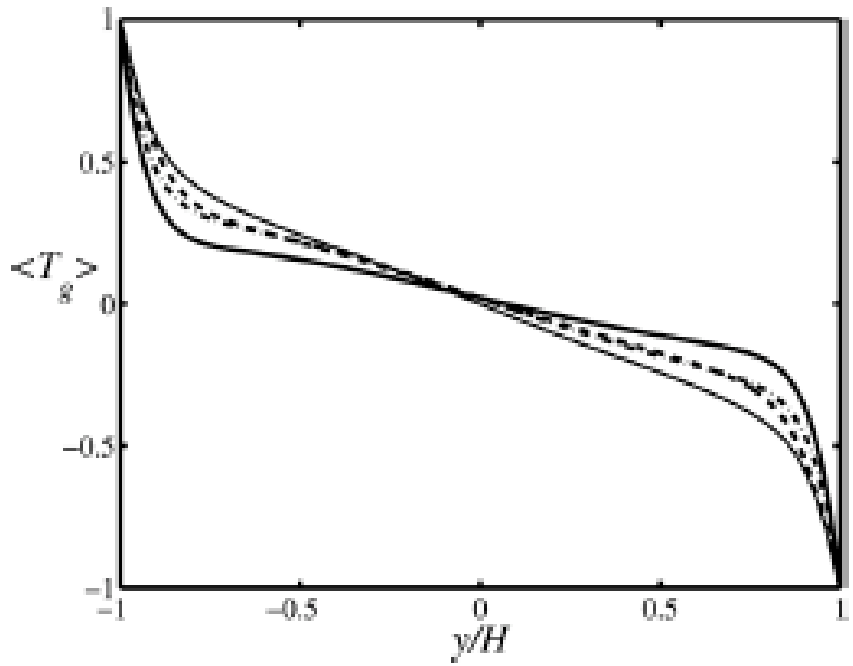
DNS of turbulent heat transfer between plates



Kuerten, et al "Turbulence modification and heat transfer enhancement by inertial particles in turbulent channel flow" Phys. Fluids 23, 123301 (2011)

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Conclusions

- Agitated suspensions have “diffusion” and “exchange” heat transfer limits.
- Granular ordering affect heat transfer.
- Granular agitation further enhances heat transfer in the exchange limit by augmenting turbulence.

Heat transfer enhancement in dense suspensions of agitated solids.
Part I: Theory. Part II: Experiments in the exchange limit

Xinglong Chen, Michel Louge

Int. J. of Heat & Mass Transfer 51 (2008) 5108–5118; 5119–5129.

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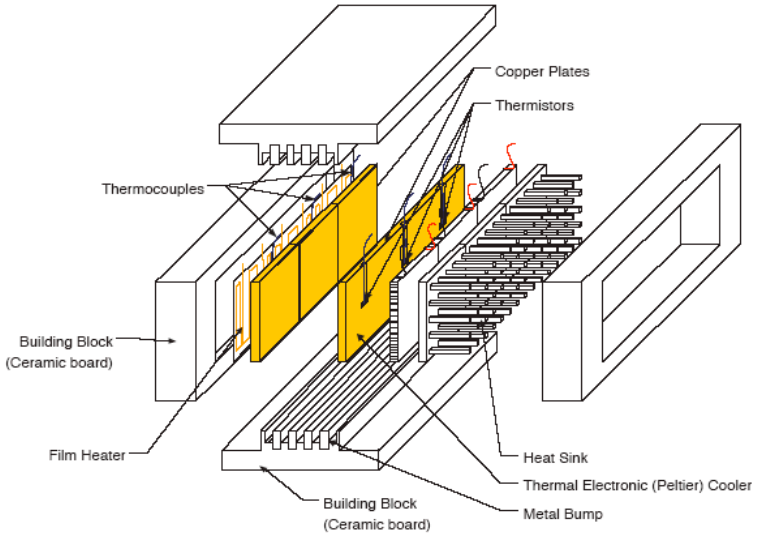
Thank you



http://grainflowresearch.mae.cornell.edu/index.html